# INSTITUTE OF DISTANCE AND OPEN LEARNING <br> Gauhati University HOMEASSIGNMENT 

## M. A./M.Sc. Mathematics

2012-2013 Session ( $2^{\text {nd }}$ Semester)

Guidelines for Submission:

1. Write your name, session, roll number, the topic selected and the title of the answer clearly on the top.
2. Each of the two topics given in each paper will be answered as two essays of not more than 500 words each. There will be negative marking for writing in excess of the word-limit
3. Each answer (essay) carries a weightage of $\mathbf{1 0}$ marks. ( 10 marks $\times 2$ essays $=20$ marks).
4. Keep a margin of about 1 inch on each side of the page.
5. You can submit the essay written in your own hand-writing on clean, foolscap sheets, or A-4 sized paper
6. In case you prefer to submit type-written answers, make sure that there are no typing errors which will deduct from the overall impression.
7. Do not submit commercially purchased answers as such a practice is deemed to be unfair.
8. Please submit your assignment by $30^{\text {th }}$ April, 2013.
9. Complex Analysis (answer any two)
$2 \times 10=20$
10. State and prove Maximum Modules theorem.
11. State and prove Laurent's theorem.
12. Find the bilinear transformation which transforms $R(z) \geq 0$ into the unit circle $|w|$ $\leq 1$.
13. Functional Analysis (answer any two) $2 \times 10=20$
14. Define Norm linear Space. State and prove Hahn Banach Thorem.
15. State and prove spectral theorem.
16. What do you mean by adjoint of an operator in a Hilbert space.Discuss some properties of it?
17. Hydrodynamics (answer any two)
$2 \times 10=20$
18. Define stream line, path line and streak lines. The velocity field at a point in fluid is given as $q=(x / t, y, 0)$. Obtain path lines, stream lines and streak lines.
19. Define source and sink in two dimensional motions. What arrangement of sources and sinks will give rise to the function $w=\log \left(z-a^{2} / z\right)$. Draw a rough sketch of the stream lines. Prove that two of the stream lines subdivide into the circle $r=a$ and axis of y .
20. What do you mean by vorticity, components of spin and vortex line? Two parallel rectilinear vortices of strengths $K_{1}$ and $K_{2}\left(K_{1}>K_{2}\right)$ are at a distance 2a apart in an infinite mass of liquid. If the vortices intersect a plane perpendicular to their length at points $A$ and $B$, show that the point on $A B$ at a distance $b$ from the midpoint on the same side of mid-point as the vortex of strength $K_{1}$, is always occupied by the same fluid element if

$$
\frac{K_{1}-K_{2}}{K_{1}+K_{2}}=\frac{b^{3}-5 a^{2} b}{a b^{2}+3 a^{3}} .
$$

204. Mathematical Methods (answer any two)
$2 \times 10=20$
205. Solve the intregal equation by successive approximation

$$
\phi(x)=\operatorname{Sin} x-\frac{x}{4}+\frac{1}{4} \int_{0}^{\pi / 2} x \phi(t) d t
$$

2. Solve the integral equation $\phi(x)=1+\int_{a}^{x} \phi(t) d t$
3. Find the Fourier integral of the function $f(x)$ where
$f(x)=\begin{array}{ll}1 & |x|<1 \\ 0 & |x|>1\end{array}$
Hence deduce that $\int_{0}^{\alpha} \frac{\operatorname{Sin} \theta}{\theta} d \theta=\frac{\pi}{2}$
4. Operation Research (answer any two)
$2 \times 10=20$
5. Solve the following problem by simplex method
$\operatorname{Max} Z=4 x_{1}+3 x_{2}$
$x_{1}+x_{2} \leq 50$
$x_{1}+2 x_{2} \geq 80$
$3 x_{1}+2 x_{2} \geq 140$
$x_{1}, x_{2} \geq 0$
6. Define OR and discuss its scope.
7. If a finite optimal feasible solution exists for the primal, there exists a finite optimal feasible solution for the dual and conversely, the values of the two objective functions are equal.
